2203000205023006
EXAMINATION FEBRUARY -MARCH 2024
BACHELOR OF SCIENCE (FIFTH SEMESTER)
MATHEMATICS-XVI (MTH-506-NUMBER THEORY-I)
LEVEL 2

[**Time: As Per Schedule]** **[Max. Marks: 50]**

Instructions:
1. Fill up strictly the following details on your answer book
   a. Name of the Examination : **BACHELOR OF SCIENCE (FIFTH SEMESTER)**
   b. Name of the Subject : **MATHEMATICS-XVI (MTH-506-NUMBER THEORY-I) LEVEL 2**
   c. Subject Code No : **2203000205023006**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. Follow usual notations.

Seat No:

Student's Signature

Q.1 Answer Any Five of the following: **10**

(1) If $a|b$ and $c|d$ then show that $ac|bd$.
(2) Assuming that gcd $(a, b) = 1$ then prove that gcd $(2a + b, a + 2b) = 1$ or 3
(3) Find lcm (119,272).
(4) Check whether the integer 2023 is a prime or composite?
(5) Solve: 81x+6y = 29.
(6) For any arbitrary integer a, verify that $2|a(a + 1)$.
(7) Verify whether integers -28, -26, -14,6,18,47,66,86,106 form a CRS (mod9).
(8) In $N = 12x41$ is divisible by 11, then find the value of x.

Q.2 Answer any two questions: **10**

(1) The given integers a and b not both of them are zero, then show that there exists integers x and y such gcd $(a, b) = ax + by$.
(2) Prove that the square of any odd integer is of the form $8k + 1$.
(3) Find integers x and y such that gcd$(1206, 6540) = 1206x + 6540y$.

Q.3 Answer any two questions: **10**

(1) If $p \geq q \geq 5; p$ and q both are prime then prove that $24|p^2 - q^2$.
(2) Determine all the positive solutions in the integers of the Diophantine equation $172x + 20y = 1000$.
(3) Find the value of x and y such that gcd $(119, 272) = 119x + 272y$.

Q.4 Answer any two questions: **10**

(1) If $n > 4$ is composite then prove that $n|(n-1)!$
(2) If $p \geq 5$ is a prime then show that $p^2 + 2$ is composite
(3) Prove that any composite number a will always have a prime divisor p such that $p \leq \sqrt{a}$.

Q.5 Answer any two questions: **10**

(1) Without performing the division, determine whether the integer 1481014095 is divisible by 9 and 11 or not.
(2) If gcd $(a, n) = 1$, then show that the integers $c, c + a, c + 2a, c + 3a, \ldots, c + (n-1)a$ form a complete set of residue modulo n, for any integer c.
(3) Prove that $111^{333} + 333^{111} \equiv 0 (mod\ 7)$.

*****